

One-loop divergences of quantum gravity coupled with scalar electrodynamics

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In non-supersymmetric covariant quantum gravity theory, for each system of gravity coupled with single field is one-loop divergent. Since adding other fields or other interactions to each system generates more possible counter-Lagrangian terms, there is room for improvement to restore renormalizability. In this paper, we consider Einstein-Maxwell fields coupled with electrically charged scalar which is the simplest model among the systems of gravity coupled with multiple fields having their own interaction and show that this system is non-renormalizable.

I. INTRODUCTION

The quantum field theory of gravitation have been developed from Feynman's pioneer works [1]. He showed that the self-consistent spin-2 quantum field theory is Einstein's general relativity. So, Einstein-Hilbert action acts as suitable action for the quantum gravity. From this action, he calculated some tree-level amplitudes such as Compton scattering and argued that every tree-level diagrams can be calculated by elementary methods. Furthermore he tried to attack one-loop diagrams and suggested fictitious quanta for unitarity of S-matrix.

After Feynman's works, Bryce DeWitt developed Feynman's results [2, 3]. He formulated manifestly covariant quantum gravity using background field method. From this formulation, he proved tree theorem and invented algorithm for S-matrix calculations containing arbitrary order radiative corrections. In this algorithm, he defined the fictitious quanta for arbitrary order. DeWitt also analyzed non-renormalizability of quantum gravity by conventional power counting method and presented tentative proposals for dealing with this situation.

An algorithm for counter-Lagrangian of one-loop diagram was introduced by G. 't Hooft [4] and this algorithm extended to include gravitation [5]. Applying this algorithm, one-loop divergences of quantum gravity coupled with scalar fields, vector fields or Yang-Mills fields were proved explicitly [5–7]. For fermionic field, the situation is quite different. Firstly, one can't use metric fields as gravitational variables. Instead of this, fermionic field has to interact with vierbein field. Furthermore, t'Hooft algorithm isn't applicable for this case because of the form of Lagrangian. S. Deser and P. van Nieuwenhuizen solved this problem by explicit calculation of the diagrams with eight external fermions and showed that Einstein-Dirac system is also non-renormalizable [8].

On the other hand, since adding other fields or other interactions to each system generates more possible counter-Lagrangian terms, there is room for improvement to restore renormalizability. In this paper, we consider Einstein-Maxwell fields coupled with electrically charged scalar which is the simplest model among the systems of gravity coupled with multiple fields having their own interaction and show that this system is non-renormalizable. For this calculation, we will quote calculation results of [5, 6].

The rest of this paper is organized as follows. In section 2, the Lagrangian for one-loop diagrams of the Einstein-Maxwell fields coupled with electrically charged scalar is obtained. In section 3, the Lagrangian for one-loop diagrams is modified in order to obtain Feynman rules and guarantee unitarity of S-matrix. In section 4, the counter-Lagrangian is calculated according to 't Hooft algorithm. Finally, in section 5, the counter-Lagrangian is reduced by applying equation of motion and it is showed that the Einstein-Maxwell fields coupled with electrically charged scalar is non-renormalizable.

II. LAGRANGIAN FOR THE ONE-LOOP DIAGRAMS

We start with gravitational field $\bar{g}_{\mu\nu}$, scalar field $\bar{\varphi}$ and electromagnetic potential \bar{A}_μ . From these variables, the Lagrangian for Einstein-Maxwell fields coupled with electrically charged scalar is

$$\mathcal{L} = -(-\bar{g})^{1/2}(\bar{R} + (D_\mu \bar{\varphi})^* \bar{g}^{\mu\nu} D_\nu \bar{\varphi} + \frac{1}{4} \bar{F}_{\mu\nu} \bar{F}_{\alpha\beta} \bar{g}^{\mu\alpha} \bar{g}^{\nu\beta}) \quad (\text{II.1})$$

where, $\sqrt{\bar{g}} = (\det \bar{g}_{\mu\nu})^{1/2}$, \bar{R} is the scalar curvature, $D_\mu \bar{\varphi} = \partial_\mu \bar{\varphi} - i \bar{A}_\mu \bar{\varphi}$ and $\bar{F}_{\mu\nu} \equiv \partial_\mu \bar{A}_\nu - \partial_\nu \bar{A}_\mu$. Note that the gravitational and electromagnetic coupling constant are omitted for a simplicity. Since complex scalar field already has kinetic term as desired form, t'Hooft algorithm is applicable for this case, unlike spin-1/2 field.

The fields $(\bar{g}_{\mu\nu}, \bar{\varphi}, \bar{A}_\mu)$ are splitted into background fields $(g_{\mu\nu}, \tilde{\varphi}, A_\mu)$ plus quantum fields $(h_{\mu\nu}, \varphi, a_\mu)$ to apply background field method,

$$\bar{g}_{\mu\nu} = g_{\mu\nu} + h_{\mu\nu} \quad (\text{II.2a})$$

$$\bar{\varphi} = \tilde{\varphi} + \varphi \quad (\text{II.2b})$$

$$\bar{A}_\mu = A_\mu + a_\mu \quad (\text{II.2c})$$

Then the equation of motion and one-loop amplitudes are calculated by expanding (II.1) with respect to quantum fields. To be specific, the Lagrangian can be expanded as follows,

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_1 + \mathcal{L}_2 + \dots \quad (\text{II.3})$$

where subindex denotes the order of quantum fields, \mathcal{L}_0 is the classical action, \mathcal{L}_1 is related to equation of motion, and \mathcal{L}_2 is one-loop diagrams part which is our interest.

The $\mathcal{L}_1, \mathcal{L}_2$ is obtained by expanding various functions of field variables with respect to quantum fields up to second order. For the scalar curvature and field strength tensor, calculation results can be found in many literatures such as [5, 6]. The kinetic terms of complex scalar field and the coupling terms between metric field, electromagnetic potential and scalar field are

$$\begin{aligned} \mathcal{L}_I \equiv \sqrt{\bar{g}} \bar{g}^{\mu\nu} D_\mu \bar{\varphi}^* D_\nu \bar{\varphi} &= (1 + \frac{1}{2} h_\alpha^\alpha - \frac{1}{4} h_\beta^\alpha h_\alpha^\beta + \frac{1}{8} (h_\alpha^\alpha)^2) (g^{\mu\nu} - h^{\mu\nu} + h^\mu_\alpha h^{\alpha\nu}) \\ &\quad ((\tilde{D}_\mu \tilde{\varphi})^* + (\tilde{D}_\mu \varphi)^* + i a_\mu \tilde{\varphi}^* + i a_\mu \varphi^*) ((\tilde{D}_\nu \tilde{\varphi}) + (\tilde{D}_\nu \varphi) - i a_\nu \tilde{\varphi} - i a_\nu \varphi) \end{aligned} \quad (\text{II.4})$$

here, we define $\tilde{D}_\mu = \partial_\mu - i A_\mu$ to distinct \bar{D}_μ . Then, the \mathcal{L}_I is expanded to the first order for the equation of motion,

$$\mathcal{L}_{1,I} = \frac{1}{2} h_\alpha^\alpha D_\mu \tilde{\varphi}^* D^\mu \tilde{\varphi} - h^{\mu\nu} D_\mu \tilde{\varphi}^* D_\nu \tilde{\varphi} - i a_\mu (\tilde{\varphi}^* \tilde{D}^\mu \tilde{\varphi} - \tilde{\varphi} \tilde{D}^\mu \tilde{\varphi}^*) + \tilde{D}_\mu \tilde{D}^\mu \tilde{\varphi} + \tilde{D}_\mu \tilde{D}^\mu \tilde{\varphi}^* \quad (\text{II.5})$$

To obtain the Lagrangian for one-loop diagram, the $\mathcal{L}_{2,I}$ part is expanded by collecting the terms containing any two quantum fields,

$$\begin{aligned} \mathcal{L}_{2,I} = & - \{ (h_\alpha^\mu h^{\alpha\nu} - \frac{1}{2} h_\alpha^\alpha h^{\mu\nu}) + (\frac{1}{8} h^2 - \frac{1}{4} h_\beta^\alpha h_\alpha^\beta) g^{\mu\nu} \} (\tilde{D}_\mu \tilde{\varphi})^* (\tilde{D}_\nu \tilde{\varphi}) \\ & - (\frac{1}{2} g^{\mu\nu} h_\alpha^\alpha - h^{\mu\nu}) \{ (\tilde{D}_\mu \varphi)^* (\tilde{D}_\nu \tilde{\varphi}) + (\tilde{D}_\mu \varphi) (\tilde{D}_\nu \tilde{\varphi})^* + i a_\mu \tilde{\varphi}^* (\tilde{D}_\nu \tilde{\varphi}) - i a_\mu \tilde{\varphi} (\tilde{D}_\nu \tilde{\varphi})^* \} \\ & - g^{\mu\nu} \{ (\tilde{D}_\mu \varphi)^* (\tilde{D}_\nu \varphi) + a_\mu a_\nu \tilde{\varphi}^* \tilde{\varphi} - i (\tilde{D}_\mu \varphi)^* a_\nu \tilde{\varphi} + i (\tilde{D}_\nu \varphi) a_\mu \tilde{\varphi}^* + i a_\mu \varphi^* (\tilde{D}_\nu \tilde{\varphi}) - i a_\nu \varphi (\tilde{D}_\mu \tilde{\varphi})^* \} \end{aligned} \quad (\text{II.6})$$

By including the scalar curvature and field strength tensor and introducing several symbols, the \mathcal{L}_2 can be written as follows,

$$\begin{aligned} \mathcal{L}_2 = & (-g)^{1/2} [-\frac{1}{2} (D_\nu h_{\alpha\beta}) P^{\alpha\beta\rho\sigma} (D^\nu h_{\rho\sigma}) + \frac{1}{2} (h_\mu - \frac{1}{2} D_\mu h)^2 - \frac{1}{2} (D_\nu a_\mu)^2 + \frac{1}{2} (D_\mu a_\nu) (D^\nu a^\mu) \\ & - (\partial_\nu \varphi)^* \partial^\nu \varphi - \varphi^* A_\nu A^\nu \varphi + i \partial_\mu \varphi^* A^\mu \varphi - i \partial_\mu \varphi A^\mu \varphi^* \\ & + \frac{1}{2} h_{\alpha\beta} (X_g + X_e + X_s)^{\alpha\beta\rho\sigma} h_{\rho\sigma} + h_{\alpha\beta} Q^{\alpha\beta\rho\sigma} D_\rho a_\sigma - a_\mu (g^{\mu\nu} \tilde{\varphi}^* \tilde{\varphi}) a_\nu \\ & + h_{\alpha\beta} B^{\alpha\beta\rho} \partial_\rho \varphi^* + i h_{\alpha\beta} B^{\alpha\beta\rho} A_\rho \varphi^* + h_{\alpha\beta} C^{\alpha\beta\rho} \partial_\rho \varphi + i h_{\alpha\beta} C^{\alpha\beta\rho} A_\rho \varphi + h_{\alpha\beta} E^{\alpha\beta\rho} a_\rho \\ & - i (\partial_\mu \varphi)^* (g^{\mu\nu} \tilde{\varphi}) a_\nu + i \partial_\mu \varphi (g^{\mu\nu} \tilde{\varphi}^*) a_\nu + i a_\nu g^{\mu\nu} (\tilde{D}_\mu \tilde{\varphi} - i A_\mu \tilde{\varphi}) \varphi^* - i a_\nu g^{\mu\nu} (\tilde{D}_\mu \tilde{\varphi}^* + i A_\mu \tilde{\varphi}^*) \varphi] \end{aligned} \quad (\text{II.7})$$

and symbols for gravitational fields in the equation are calculated from the expansion as listed in below,

$$P^{\alpha\beta\rho\sigma} = \frac{1}{2} g^{\alpha\rho} g^{\beta\sigma} - \frac{1}{4} g^{\alpha\beta} g^{\rho\sigma} \quad (\text{II.8a})$$

$$X_g^{\alpha\beta\rho\sigma} = P^{\alpha\beta\rho\sigma} R - g^{\alpha\rho} R^{\beta\sigma} + g^{\alpha\beta} R^{\rho\sigma} + R^{\alpha\rho\beta\sigma} \quad (\text{II.8b})$$

$$X_e^{\alpha\beta\rho\sigma} = P^{\alpha\beta\rho\sigma} \frac{1}{4} F^2 - \frac{1}{2} F^{\alpha\rho} F^{\beta\sigma} - g^{\alpha\rho} F_2^{\beta\sigma} + \frac{1}{2} g^{\alpha\beta} F_2^{\rho\sigma} \quad (\text{II.8c})$$

$$X_s^{\alpha\beta\rho\sigma} = -2 g^{\sigma\beta} (\tilde{D}^\rho \tilde{\varphi})^* (\tilde{D}^\alpha \tilde{\varphi}) + g^{\alpha\beta} (\tilde{D}^\rho \tilde{\varphi})^* (\tilde{D}^\sigma \tilde{\varphi}) - \frac{1}{4} g^{\alpha\beta} g^{\rho\sigma} (\tilde{D}^\nu \tilde{\varphi})^* (\tilde{D}^\nu \tilde{\varphi}) + \frac{1}{2} g^{\sigma\beta} g^{\rho\alpha} (\tilde{D}^\nu \tilde{\varphi})^* (\tilde{D}^\nu \tilde{\varphi}) \quad (\text{II.8d})$$

and symbols for gravitational field coupled to Maxwell field or scalar field in the equation are

$$Q^{\alpha\beta\rho\sigma} = 2g^{\alpha\rho}F^{\beta\sigma} - \frac{1}{2}g^{\alpha\beta}F^{\rho\sigma} \quad (\text{II.9a})$$

$$B^{\alpha\beta\rho} = -\frac{1}{2}g^{\alpha\beta}\tilde{D}^\rho\tilde{\varphi} + g^{\rho\alpha}\tilde{D}^\beta\tilde{\varphi} \quad (\text{II.9b})$$

$$C^{\alpha\beta\rho} = -\frac{1}{2}g^{\alpha\beta}(\tilde{D}^\rho\tilde{\varphi})^* + g^{\rho\alpha}(\tilde{D}^\beta\tilde{\varphi})^* \quad (\text{II.9c})$$

$$E^{\alpha\beta\rho} = -\frac{1}{2}ig^{\alpha\beta}\tilde{D}^\rho\tilde{\varphi} + ig^{\rho\alpha}\tilde{D}^\beta\tilde{\varphi} + \frac{1}{2}ig^{\alpha\beta}(\tilde{D}^\rho\tilde{\varphi})^* - ig^{\rho\alpha}(\tilde{D}^\beta\tilde{\varphi})^* \quad (\text{II.9d})$$

where $F_2^{\mu\nu} \equiv F_2^{\nu\mu} \equiv F_\alpha^\mu F^{\nu\alpha}$ and $F_2^\mu \equiv F^2$.

III. GAUGE FIXING AND GHOST

In this section, quadratic part of our Lagrangian is modified to obtain the Feynman rules for S-matrix. First, consider the following gauge transformations,

$$h'_{\mu\nu} = h_{\mu\nu} + (g_{\mu\alpha} + \kappa h_{\mu\alpha})\eta^\alpha_{,\nu} + (g_{\alpha\nu} + \kappa h_{\alpha\nu})\eta^\alpha_{,\mu} + \eta^\alpha(g_{\mu\nu} + \kappa h_{\mu\nu})_{,\alpha} \quad (\text{III.1a})$$

$$a'_\mu = a_\mu + (A_\alpha + \kappa a_\alpha)\eta^\alpha_{,\mu} + \eta^\alpha(A_\mu + \kappa a_\mu)_{,\alpha} + \partial_\mu\Lambda \quad (\text{III.1b})$$

where $x^\alpha - x'^\alpha = \kappa\eta^\alpha$ and $\partial_\mu\Lambda$ is usual electromagnetic gauge. Here, these transformations are rewritten in terms of covariant derivatives D_α as follows,

$$h'_{\mu\nu} = h_{\mu\nu} + (g_{\mu\alpha}D_\nu + g_{\nu\alpha}D_\mu)\eta^\alpha + \kappa[(h_{\mu\alpha}D_\nu + h_{\nu\alpha}D_\mu)\eta^\alpha + \eta^\alpha D_\alpha h^{\mu\nu}] \quad (\text{III.2a})$$

$$a'_\mu = a_\mu + (A_\alpha D_\mu \eta^\alpha + \eta^\alpha D_\alpha A_\mu) + \kappa(a_\alpha D_\mu \eta^\alpha + \eta^\alpha D_\alpha a_\mu) + D_\mu\Lambda \quad (\text{III.2b})$$

To express (III.2b) in terms of $F_{\mu\nu}$, let us take $\Lambda = -\eta^\alpha A_\alpha + \eta^5$ then,

$$a'_\mu = a_\mu + \eta^\alpha F_{\alpha\mu} + D_\mu\eta^5 + \kappa(a_\alpha D_\mu \eta^\alpha + \eta^\alpha D_\alpha a_\mu) \quad (\text{III.3})$$

Our original action is then invariant under these transformations,

$$\int d^4x' \mathcal{L}(\bar{g}', \bar{A}', \bar{\varphi}', (\bar{\varphi}^*)') = \int d^4x \mathcal{L}(\bar{g}, \bar{A}, \bar{\varphi}, \bar{\varphi}^*) \quad (\text{III.4})$$

To obtain Feynman rules, it is needed to choose gauge fixing terms $-\frac{1}{2}C_\mu^2$ for gravitational fields and vector fields respectively and include ghost Lagrangian in our calculations. From the form of (II.7), one can choose C_μ as follows,

$$C_a = (-g)^{\frac{1}{4}}e_\alpha^\mu(h_\mu - \frac{1}{2}D_\mu h) \quad (\text{III.5a})$$

$$C_5 = (-g)^{\frac{1}{4}}D_\mu a^\mu \quad (\text{III.5b})$$

where e_α^μ is a square root of a metric field which is called a vierbein field.

With these gauge fixing terms, we can finally write quadratic Lagrangian for non-ghost parts :

$$\begin{aligned} \mathcal{L}_{NG} = & (-\bar{g})^{1/2}[-\frac{1}{2}(D_\nu h_{\alpha\beta})P^{\alpha\beta\rho\sigma}(D^\nu h_{\rho\sigma}) - \frac{1}{2}(D_\nu a_\mu)^2 - (\partial_\nu\varphi)^*\partial^\nu\varphi - \varphi^*A_\nu A^\nu\varphi + i\partial_\mu\varphi^*A^\mu\varphi - i\partial_\mu\varphi A^\mu\varphi^* \\ & + \frac{1}{2}h_{\alpha\beta}(X_g + X_e + X_s)^{\alpha\beta\rho\sigma}h_{\rho\sigma} + h_{\alpha\beta}Q^{\alpha\beta\rho\sigma}D_\rho a_\sigma - a_\mu(-\frac{1}{2}R^{\mu\nu} + g^{\mu\nu}\tilde{\varphi}^*\tilde{\varphi})a_\nu \\ & + h_{\alpha\beta}B^{\alpha\beta\rho}\partial_\rho\varphi^* + ih_{\alpha\beta}B^{\alpha\beta\rho}A_\rho\varphi^* + h_{\alpha\beta}C^{\alpha\beta\rho}\partial_\rho\varphi + ih_{\alpha\beta}C^{\alpha\beta\rho}A_\rho\varphi + h_{\alpha\beta}E^{\alpha\beta\rho}a_\rho \\ & - i(\partial_\mu\varphi)^*(g^{\mu\nu}\tilde{\varphi})a_\nu + i\partial_\mu\varphi(g^{\mu\nu}\tilde{\varphi}^*)a_\nu + ia_\nu g^{\mu\nu}(\tilde{D}_\mu\tilde{\varphi} - iA_\mu\tilde{\varphi})\varphi^* - ia_\nu g^{\mu\nu}(\tilde{D}_\mu\tilde{\varphi}^* + iA_\mu\tilde{\varphi}^*)\varphi] \end{aligned} \quad (\text{III.6})$$

Here, the Ricci identity is used

$$\begin{aligned} (D_\alpha D_\beta - D_\beta D_\alpha)A^\mu &= R_{\gamma\alpha\beta}^\mu A^\gamma \\ (D_\mu D_\beta - D_\beta D_\mu)A^\mu &= -R_{\mu\beta}A^\mu \end{aligned} \quad (\text{III.7})$$

On the other hand, the ghost Lagrangian \mathcal{L}_G can be calculated by subjecting C_μ to the gauge transformations (III.2a),(III.3). From (III.5a) and (III.5b), we find

$$\mathcal{L}_G = (-g)^{\frac{1}{4}}(\phi^{*\alpha}, \chi^*) \begin{pmatrix} e_{\alpha\beta} D_\nu D^\nu - R_{\alpha\beta} & 0 \\ -(D^\lambda F_{\lambda\beta}) - F_{\lambda\beta} D^\lambda & D_\nu D^\nu \end{pmatrix} \begin{pmatrix} \phi^\beta \\ \chi \end{pmatrix} \quad (\text{III.8})$$

where, ϕ^α is a vector ghost and χ is a scalar ghost.

IV. COUNTER-LAGRANGIAN

In this section, counter-Lagrangian for the Einstein-Maxwell fields coupled with electrically charged scalar is derived using above results. Our Lagrangian should be transformed to fit into the following scalar form to apply 't Hooft algorithm:

$$\mathcal{L} = (-g)^{1/2}(\phi_i^* D_\nu D^\nu \phi_i + 2\phi_i^* N^{\mu,ij} \partial_\mu \phi_j + \phi_i^* M^{ij} \phi_j) \quad (\text{IV.1})$$

This is accomplished by following procedure. First, we introduce complex fields $h \equiv (h_1 + ih_2)2^{1/2}$ and $a \equiv (a_1 + ia_2)2^{1/2}$ where h_1, h_2, a_1, a_2 are identical with h, a . Since we double our field variables, it should be counted at the end of calculation. To fit into the (IV.1), integral by parts should be performed for the terms containing $D\phi^*$ as follows,

$$hQ(Da)^* = -DhQa^* - h(DQ)a^* \quad (\text{IV.2a})$$

$$hA(D\varphi)^* = -DhA\varphi^* - h(DA)\varphi^* \quad (\text{IV.2b})$$

$$a\tilde{\varphi}(D\varphi)^* = -Da\tilde{\varphi}\varphi^* - a(D\tilde{\varphi})\varphi^* \quad (\text{IV.2c})$$

$$\varphi A(D\varphi)^* = -D\varphi A\varphi^* - \varphi(DA)\varphi^* \quad (\text{IV.2d})$$

Second, we replace $h_{\alpha\beta}^* P^{\alpha\beta\rho\sigma} \rightarrow h_{\rho\sigma}^*$ and $a_\alpha^* g^{\alpha\beta} \rightarrow a_\beta^*$ which are not change counter Lagrangian according to lemma in [5]. And finally, double-derivative terms are expressed in terms of \tilde{D} which is not work on explicit field indices as follows,

$$h_{\alpha\beta}^* D_\nu D^\nu h_{\alpha\beta} = h_{\alpha\beta}^* \tilde{D}_\nu \tilde{D}^\nu h_{\alpha\beta} + 2h_{\alpha\beta}^* \mathcal{N}^{\mu\rho\sigma}_{\alpha\beta} \tilde{D}_\mu h_{\rho\sigma} + h_{\alpha\beta}^* \mathcal{T}^{\rho\sigma}_{\alpha\beta} h_{\alpha\beta} \quad (\text{IV.3a})$$

$$a_\alpha^* D_\nu D^\nu a_\alpha = a_\alpha^* \tilde{D}_\nu \tilde{D}^\nu a_\alpha + 2a_\alpha^* n^{\mu\beta}_{\alpha} \tilde{D}_\mu a_\beta + a_\alpha^* \tau_\alpha^\beta a_\beta \quad (\text{IV.3b})$$

where

$$\mathcal{N}^{\mu\rho\sigma}_{\alpha\beta} = -2g^{\mu\lambda} \Gamma_{\lambda\alpha}^\rho \delta_\beta^\sigma \quad (\text{IV.4a})$$

$$\mathcal{T}^{\rho\sigma}_{\alpha\beta} = (D_\mu \mathcal{N}^\mu + \mathcal{N}_\mu \mathcal{N}^\mu)_{\alpha\beta}^{\rho\sigma} \quad (\text{IV.4b})$$

$$n^{\mu\beta}_{\alpha} = -g^{\mu\lambda} \Gamma_{\lambda\alpha}^\rho \quad (\text{IV.4c})$$

$$\tau_\alpha^\beta = (D_\mu n^\mu + n_\mu n^\mu)_\alpha^\beta \quad (\text{IV.4d})$$

Applying these formula and performing some algebra, the form of (IV.1) is obtained in terms of 10+4+1 independent complex fields $\phi_i = (h_{\mu\nu}, a_\mu, \varphi)$ with

$$N^\mu_{NG} = \begin{pmatrix} \mathcal{N}^{\mu\rho\sigma}_{\alpha\beta} & (P^{-1} \frac{1}{2} Q^\mu)_{\alpha\beta}^\delta & \frac{1}{2} P^{-1} C^{\alpha\beta\mu} \\ -\frac{1}{2} g_{\gamma\lambda} Q^{\rho\sigma\mu\lambda} & n^{\mu\delta}_\gamma & \frac{1}{2} i \delta_\gamma^\mu \tilde{\varphi}^* \\ -\frac{1}{2} B^{\alpha\beta\mu} & \frac{1}{2} i g^{\mu\gamma} \tilde{\varphi} & -i A^\mu \end{pmatrix} \quad (\text{IV.5a})$$

$$M_{NG} = \begin{pmatrix} P^{-1}(X_g + X_s + X_e) + \mathcal{T} & P^{-1} E^{\alpha\beta\rho} & i P^{-1} C^{\alpha\beta\rho} A_\rho \\ -g_{\gamma\lambda} \partial_\mu Q^{\rho\sigma\mu\lambda} + g_{\gamma\lambda} E^{\alpha\beta\lambda} & R_\gamma^\delta - 2\delta_\gamma^\delta \tilde{\varphi}^* \tilde{\varphi} + \tau_\gamma^\delta & -i \tilde{D}_\nu \tilde{\varphi}^* + A_\nu \tilde{\varphi}^* \\ -\partial_\rho B^{\alpha\beta\rho} + i A_\rho B^{\alpha\beta\rho} & 2i \tilde{D}^\nu \tilde{\varphi} & -A_\nu A^\nu - i \partial_\nu A^\nu \end{pmatrix} \quad (\text{IV.5b})$$

Since the ghost Lagrangian already has desired form, the N_G^μ and M_G is written directly as follows:

$$N^\mu_G = \begin{pmatrix} n^{\mu\beta}_\alpha & 0 \\ -\frac{1}{2} F_{\lambda\beta} & 0 \end{pmatrix} \quad (\text{IV.6a})$$

$$M_G = \begin{pmatrix} -R_\alpha^\beta + \tau_\alpha^\beta & 0 \\ -D^\lambda F_{\lambda\alpha}^\beta & 0 \end{pmatrix} \quad (\text{IV.6b})$$

Note that the factor $(-g)^{1/4}e_{\alpha\beta}$ is absorbed into ϕ^* and also \tilde{D}_ν is applied as non-ghost case. Each element of matrices M_μ, N corresponds to vertex among two quantum fields and one external field.

Now, we ready to calculate counter Lagrangian. The counter-Lagrangian is calculated by the following formula with above N^μ and M :

$$\Delta\mathcal{L} = \frac{1}{\epsilon}(-g)^{1/2}\{\text{tr}[\frac{1}{12}Y_{\mu\nu}Y^{\mu\nu} + \frac{1}{2}X^2 + \frac{1}{60}(R_{\mu\nu}R^{\mu\nu} - \frac{1}{3}R^2)]\} \quad (\text{IV.7})$$

where

$$Y_{\mu\nu} = \partial_\mu N_\nu - \partial_\nu N_\mu + N_\mu N_\nu - N_\nu N_\mu \quad (\text{IV.8a})$$

$$X = M - D_\mu N^\mu - N_\mu N^\mu - \frac{1}{6}R \quad (\text{IV.8b})$$

Note that each diagonal element of matrices $Y_{\mu\nu}Y^{\mu\nu}, X^2$ corresponds up to one-loop diagram with 4-legs. There are several comments before this calculation. First, the trace is to be taken over the 15 independent fields ($10(h_{\mu\nu})+1(\varphi)+4(a_\mu)$) for the non-ghost parts, and 5 independent fields ($4(\phi^a)+1(\chi)$) for the ghost parts. For example when we calculate the last part of (IV.7) for the non-ghost parts, we should multiply 15 to $\frac{1}{60}(R_{\mu\nu}R^{\mu\nu} - \frac{1}{3}R^2)$. Second for the non-ghost parts, the doubling should be undo by dividing 2 and for the ghost parts it is need to add extra minus sign.

For the non-ghost Lagrangian, explicit $Y_{\mu\nu}$ and X are

$$Y_{\mu\nu} = \begin{pmatrix} \partial_\mu \mathcal{N}_\nu - \partial_\nu \mathcal{N}_\mu + \mathcal{V}_{\mu\nu} + \mathcal{W}_{\mu\nu} & \partial_\mu Q_\nu - \partial_\nu Q_\mu & 3(\partial_\mu(\tilde{D}_\nu \tilde{\varphi})^* - \partial_\nu(\tilde{D}_\mu \tilde{\varphi})^*) \\ -\frac{1}{2}\partial_\mu Q_\nu + \frac{1}{2}\partial_\nu Q_\mu & \partial_\mu n_\nu - \partial_\nu n_\mu + \mathcal{V}_{\mu\nu} & \frac{1}{2}i\partial_\mu g_{\nu\gamma}\tilde{\varphi}^* - \frac{1}{2}i\partial_\nu g_{\mu\gamma}\tilde{\varphi}^* \\ -\frac{1}{2}\partial_\mu B_\nu + \frac{1}{2}\partial_\nu B_\mu & \frac{1}{2}i\partial_\mu \delta_\nu^\gamma \tilde{\varphi} - \frac{1}{2}i\partial_\nu \delta_\mu^\gamma \tilde{\varphi} & -iF_{\mu\nu} + \mathcal{W}_{\mu\nu} \end{pmatrix} \quad (\text{IV.9a})$$

$$X = \begin{pmatrix} \mathcal{H} & 3i(\tilde{D}_\nu \tilde{\varphi} - (\tilde{D}_\nu \tilde{\varphi})^*) - \partial_\mu Q^\mu & 3(\tilde{D}_\mu \tilde{D}^\mu \tilde{\varphi})^* \\ g_{\gamma\lambda}E^{\alpha\beta\lambda} - \frac{1}{2}\partial_\mu Q^\mu & \mathcal{I} & -2i(\tilde{D}_\nu \tilde{\varphi})^* - \frac{1}{2}i\partial_\nu \tilde{\varphi}^* \\ \tilde{D}_\rho B^{\alpha\beta\rho} - \frac{3}{2}\partial_\rho B^{\alpha\beta\rho} & +2i\tilde{D}_\nu \tilde{\varphi} + \frac{1}{2}i\partial_\nu \tilde{\varphi} & \mathcal{J} \end{pmatrix} \quad (\text{IV.9b})$$

where the symbols in $Y_{\mu\nu}$ are

$$\mathcal{V}_{\mu\nu} = -\frac{1}{2}Q_\mu Q_\nu + \frac{1}{2}Q_\nu Q_\mu \quad (\text{IV.10a})$$

$$\mathcal{W}_{\mu\nu} = -\frac{1}{2}C_\mu B_\nu + \frac{1}{2}C_\nu B_\mu \quad (\text{IV.10b})$$

and the symbols in X are

$$\mathcal{H} = P^{-1}(X_g + X_e + X_s) + \frac{1}{2}Q_\mu Q^\mu + \frac{1}{2}C_\mu B^\mu - \frac{1}{6}R \quad (\text{IV.11a})$$

$$\mathcal{I} = R^{\alpha\beta} - 2g^{\alpha\beta}\tilde{\varphi}\tilde{\varphi}^* + \frac{1}{2}Q_\mu Q^\mu + \frac{1}{4}\tilde{\varphi}\tilde{\varphi}^* - \frac{1}{6}R \quad (\text{IV.11b})$$

$$\mathcal{J} = -A_\nu A^\nu - i\partial_\nu A^\nu + \frac{1}{2}C_\mu B^\mu + \frac{1}{4}\tilde{\varphi}\tilde{\varphi}^* - \frac{1}{6}R \quad (\text{IV.11c})$$

Note that $P^{-1}Q = 2Q, P^{-1}X_g = 2X_g$ and $P^{-1}C = 3(\tilde{D}_\nu \tilde{\varphi})^*$ and for calculating $Y_{\mu\nu}$ and X , off diagonal parts of NN-term are omitted by dimensionality and gauge covariance in background fields. After inserting these to (IV.7), counter Lagrangian for non-ghost fields is written as follows :

$$\begin{aligned} \Delta\mathcal{L}_{NG} = & (-g)^{1/2}[\frac{17}{24}R_{\mu\nu}R^{\mu\nu} + \frac{13}{48}R^2 + \frac{1}{6}R_{\mu\nu}T^{\mu\nu} + \frac{13}{24}T_{\mu\nu}^2 + \frac{1}{6}(D^\alpha F_{\alpha\beta})^2 \\ & + ((\tilde{D}_\mu \tilde{\varphi})^* g^{\mu\nu} \tilde{D}_\nu \tilde{\varphi})^2 - \frac{1}{6}R((\tilde{D}_\mu \tilde{\varphi})^* g^{\mu\nu} \tilde{D}_\nu \tilde{\varphi})] \end{aligned} \quad (\text{IV.12})$$

here, $T_{\mu\nu} \equiv F_{\mu\alpha}F_\nu^\alpha - \frac{1}{4}g_{\mu\nu}F^{\alpha\beta}F_{\alpha\beta}$ is the energy-momentum tensor for EM-potential. For the ghost Lagrangian, trace of the matrices $Y_{\mu\nu}^2$ and X^2 are

$$Y_{\mu\nu}^2 = -4R_{\mu\nu}^2 + R^2 \quad (\text{IV.13a})$$

$$X^2 = R_{\mu\nu}^2 + (\frac{1}{3} + \frac{5}{36})R^2 \quad (\text{IV.13b})$$

After inserting these to (IV.7), counter Lagrangian for ghost fields is written as follows :

$$\Delta\mathcal{L}_G = -(-g)^{1/2}[\frac{1}{4}R_{\mu\nu}^2 + \frac{7}{24}R^2] \quad (\text{IV.14})$$

The total counter Lagrangian is the sum of (IV.12) and (IV.14)

$$\begin{aligned} \Delta\mathcal{L}_{tot} = & (-g)^{1/2}[\frac{11}{24}R_{\mu\nu}R^{\mu\nu} - \frac{1}{48}R^2 + \frac{1}{6}R_{\mu\nu}T^{\mu\nu} + \frac{13}{24}T_{\mu\nu}^2 + \frac{1}{6}(D^\alpha F_{\alpha\beta})^2 \\ & + ((\tilde{D}_\mu\tilde{\varphi})^* g^{\mu\nu} \tilde{D}_\nu\tilde{\varphi})^2 - \frac{1}{6}R((\tilde{D}_\mu\tilde{\varphi})^* g^{\mu\nu} \tilde{D}_\nu\tilde{\varphi})] \end{aligned} \quad (\text{IV.15})$$

V. EQUATION OF MOTION

In this section, counter-Lagrangian is reduced by applying equation of motion. The equation of motion is obtained by requiring that the action is stationary with respect to variations. With (II.5), the equation of motion for each the quantum fields $h_{\mu\nu}, a_\mu, \varphi$ are

$$\tilde{D}_\mu\tilde{D}^\mu\tilde{\varphi} = 0, \quad \tilde{D}_\mu\tilde{D}^\mu\tilde{\varphi}^* = 0 \quad \text{:Klein-Gordon equation. (V.1a)}$$

$$D_\alpha F^{\alpha\beta} = i(\tilde{\varphi}^*\tilde{D}^\mu\tilde{\varphi} - \tilde{\varphi}\tilde{D}^\mu\tilde{\varphi}^*) \quad \text{:Maxwell equation. (V.1b)}$$

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = -\frac{1}{2}(2\tilde{D}_\mu\tilde{\varphi}^*\tilde{D}_\nu\tilde{\varphi} - g_{\mu\nu}\tilde{D}^\alpha\tilde{\varphi}^*\tilde{D}_\alpha\tilde{\varphi} + F_{\mu\alpha}F_\nu^\alpha - \frac{1}{4}g_{\mu\nu}F^{\alpha\beta}F_{\alpha\beta}) \quad \text{:Einstien equation. (V.1c)}$$

To express $R_{\mu\nu}$ and R separately, let us take the trace of (V.1c) then

$$R = -\tilde{D}_\alpha\tilde{\varphi}^*D^\alpha\tilde{\varphi} - \frac{1}{4}F_{\alpha\beta}F^{\alpha\beta} \quad (\text{V.2})$$

Inserting it back to (V.1c), we find

$$R_{\mu\nu} = -\tilde{D}_\mu\tilde{\varphi}^*D_\nu\tilde{\varphi} - \frac{1}{2}F_{\mu\alpha}F_\nu^\alpha \quad (\text{V.3})$$

Substituting these to counter Lagrangian (IV.15), it is reduced to

$$\Delta\mathcal{L}_{tot} = (-g)^{1/2}[\frac{11}{24}R_{\mu\nu}R^{\mu\nu} + \frac{55}{48}R^2 + \frac{1}{6}R_{\mu\nu}T^{\mu\nu} + \frac{13}{24}T_{\mu\nu}^2 + \frac{1}{6}(D^\alpha F_{\alpha\beta})^2 + \frac{13}{24}RF^2 + \frac{1}{16}(F^2)^2] \quad (\text{V.4a})$$

Unlike Einstein-Klein-Gordon or Einstein-Maxwell case, $\Delta\mathcal{L}$ is not expressed by the form of $aR_{\mu\nu}R^{\mu\nu} + bR^2$, since equations of motion are coupled. But we can see easily that there are no more cancellation with these conditions. So the theory of the Einstein-Maxwell fields coupled with electrically charged scalar is non-renormalizable.

VI. CONCLUSIONS

In this paper, the counter-Lagrangian of Einstein-Maxwell fields coupled with electrically charged scalar is derived in non-supersymmetric covariant theory. By adding scalar field to Einstein-Maxwell fields, there are more possible counter-Lagrangian terms, but these do not remove the divergent terms. Here, scalar field is added to Einstein-Maxwell fields because of a simplicity of the model. However, there are still possibility to remove the divergent terms by considering more complex systems in non-supersymmetric covariant theory. On the other hand, supersymmetry provides the physical principle to add other fields, namely supersymmetric partners. Furthermore, in that case, there are miraculous cancellations in loop calculations.

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